

Cognitive Abilities Test™
Practice Activities
Teacher Guide

Form 7

Quantitative
Tests

Level 8

CogAT®

Test 4: Number Analogies, Level 8

Part 1: Overview of the Number Analogies

An analogy draws parallels between objects or ideas. Analogies can be simple (“Two is one more than one as three is one more than two”) or complex (“Friendships are like glass. Once broken, they are difficult to restore”). Successful learners habitually reason by analogy. Good analogies allow them to use what they already know when they are trying to understand or remember new ideas. Reasoning by analogy requires attending carefully to the ways in which two things are similar. Then this relationship is mapped onto something new.

The Number Analogies test requires the student to understand the relationship between the two pictures of objects in the top row of each question and then select an answer that makes the two pictures in the bottom row of the question go together in the same way. Performance on this test predicts mathematics achievement because discovering quantitative patterns and relationships is at the core of learning mathematics.

In this test, students are asked to solve problems that look like this:

What rule can tell us what happened between the first and second boxes?

Apply the same rule to figure out how many balls should go in the box with the question mark.

When practicing the Number Analogies questions, encourage students to use these strategies.

- Think of (and say silently) a rule that describes what happens between the first and second pictures. For example, the second picture has one less item than the first picture.
- Apply the rule to the first picture in the bottom row to determine how the missing picture should appear. Then look for this picture in the answer choices.
- Test the rule on each answer choice, eliminating answer choices that do not fit the rule.

Students at this level tend to make the following common mistakes.

- Students may have a tendency to select an answer choice based on number or appearance only. For example, in the sample question above, a student might pick the second answer choice because it looks like the second picture in the top row.
- Students might ignore the direction of the arrow. For example, a student may pick the third answer choice in the sample question above so both rows have one picture with four things and one picture with five things.

Part 2: Number Analogies Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in *blue italics*: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how to do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.

(Make sure each student has a practice booklet. Then **SAY**:)

*Open your practice booklet to page 1. You should be on the page with **bicycles** across the top.*

(Check that all students have the correct page.)

P1

Let's do the first practice question.



(Hold up your copy of the booklet. Point to the big box that has pictures inside of it as you **SAY**:)

The big box has three pictures. We must decide which answer picture goes in the box with the question mark.

Look at the two pictures in the top row of the big box. The arrow between them means that the two pictures in the top row go together in some way.

(Point to the pictures of stars in the top row as you **SAY**:)

How do the pictures of stars change from the first box to the second box?

(Encourage responses.)

In the first box, there is one star, and in the second box there are two stars.

So what is the rule for this question?

(Encourage responses.)

The rule is to add one to the number of things in the first box.

(Note that the rule could also be to double the items in the first box. Here both rules give the same answer.)

Now look at the first picture of the bug in the bottom row of the big box.

(Point to the bug and the box with the question mark as you **SAY**.)

The bug must change in the same way that the star changed. How many bugs should go in the box with the question mark?

(Encourage responses.)

There should be two bugs in the box with the question mark.

(If any students seem confused, ask them to draw the two bugs in the box with the question mark. Then point to the answer pictures as you **SAY**.)

Which answer picture should we choose?

(Encourage responses.)

There should be two bugs in the box with the question mark. Only one picture has two bugs.

Fill in the circle below the picture with two bugs to show that it is the best answer.

(Make sure that all students have filled in the second circle.)

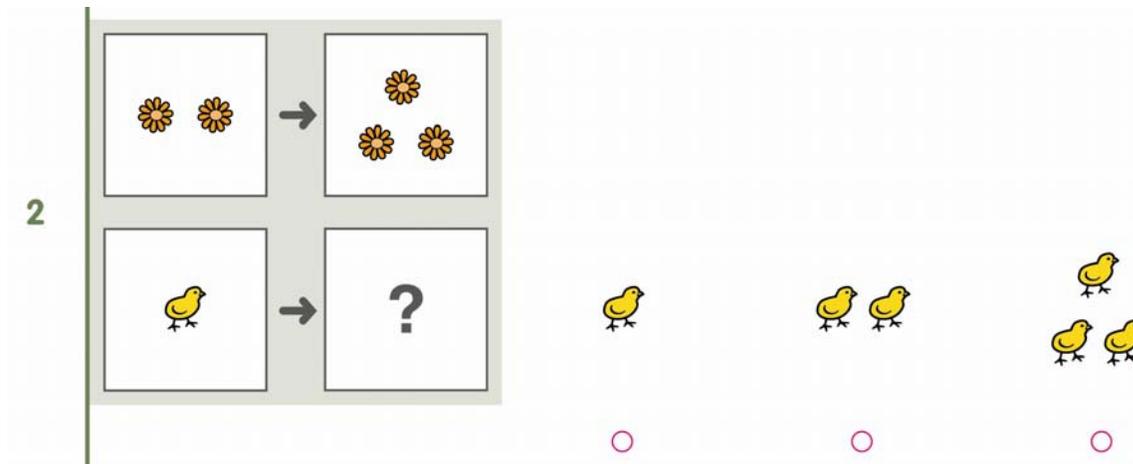
Would it be OK if the bugs were side by side instead of on top of each other?

(Encourage responses.)

*Yes, two bugs side by side would be a better answer. A picture of two bugs side by side would show the right number **and** look the same as the two pictures of stars in the top row. But this was not one of the choices. Only the second answer choice had two bugs, so it is the best answer.*

P2

Let's do the next practice question.



Look at the top row of the question.

(Point to the pictures of flowers in the top row as you **SAY**.)

How do the pictures of flowers change from the first box to the second box?

(Encourage responses.)

The first picture has two flowers and the second picture has three flowers.

So what is the rule for this question?

(Encourage responses.)

The rule for this question is the same as the rule for the first question. The rule is to add one to the number of things in the first box.

Now look at the bottom row of the big box.

(Point to the chick and the question mark as you **SAY**.)

The pictures in the bottom row must follow the same rule as the pictures in the top row. How many chicks should there be in the box with the question mark?

(Encourage responses.)

How many chicks are in the first box?

(Encourage responses.)

What is the rule for this question?

(Encourage responses. Then point to the answer choices as you **SAY**.)

So which answer picture goes in the box with the question mark?

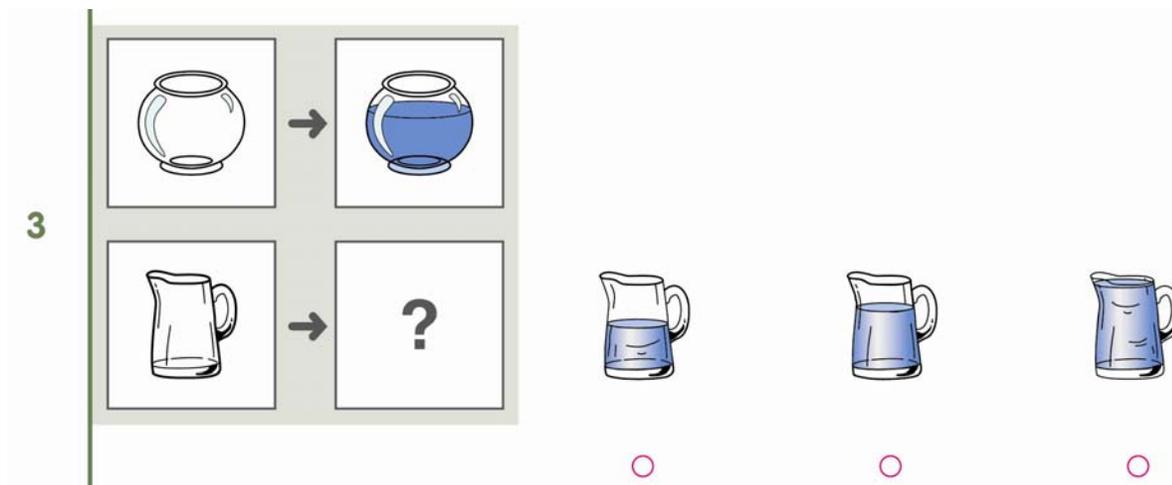
(Encourage responses.)

The picture with two chicks is the correct answer. Fill in the second circle, the circle under the picture of two chicks, to show that it is the correct answer.

(Check to make sure that all students have filled in the second circle. Be sure that students understand why the third answer choice is incorrect, even though it looks like the picture with three flowers. Ask “What is the rule?” and show again how it is applied. Then ask “What if the first box had four chicks?”)

P3

Let's do the next practice question.



Look at the top row of the big box.

(Point to the pictures of fish bowls in the top row as you **SAY**.)

How do the pictures of fish bowls change from the first box to the second box?

(Encourage responses.)

The first fish bowl is empty. How full of water is the second fish bowl? Is it half full? Is it all the way full?

(Encourage students to express this measurement in words.)

The second fish bowl is more than half full, but not all the way full. Now look at the bottom row.

(Point to the pitcher and the question mark as you **SAY**.)

The two pitchers in the bottom row must follow the same rule as the fish bowls in the top row.

(Point to the answer choices as you **SAY**.)

Which pitcher of water is the correct answer?

(Encourage responses. Then point to the second answer picture.)

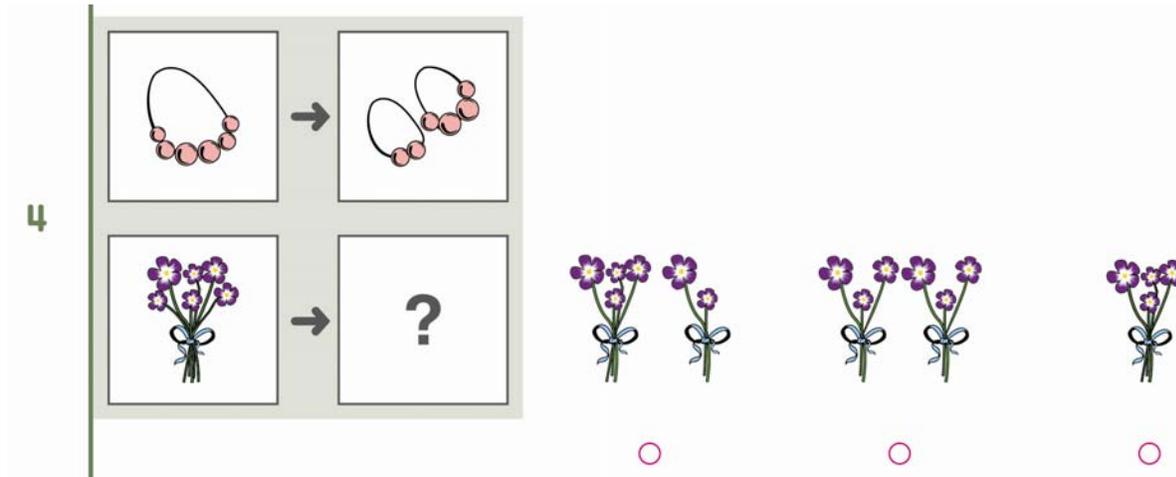
The pitcher should be more than half full but not filled to the top. The second answer choice is the correct answer.

(Check to make sure that all students have filled in the second circle. The first answer choice cannot be the answer because the water fills only half, and the pitcher in the third answer choice is completely filled.)

Turn to the next page. You should be on the page with the **combs** across the top.

P4

Let's do the next practice question.



Look at the necklaces in the top row.

(Point to the pictures of necklaces in the top row as you **SAY**.)

How do the pictures of necklaces change from the first box to the second box?

(Encourage responses.)

How many beads are on the first necklace?

(Encourage responses.)

The first necklace has six beads. What about the second picture?

(Encourage responses.)

The second picture has two beads on the first necklace and four beads on the second necklace.

What rule can we use to show what happened to the necklace between the first and second box?

(Encourage responses.)

The six beads on the necklace in the first picture were shared between two necklaces in the second picture. One necklace has two beads and the other has four beads.

Now look at the bottom row.

(Point to the flowers and the question mark as you **SAY**.)

The pictures of the bouquets, or bunches, of flowers must follow the same rule as the pictures of the necklaces in the top row. How many flowers are in the first bouquet or bunch?

(Encourage responses.)

There are six flowers in the first bouquet.

(Point to the answer choices as you **SAY**.)

Think about how the answer picture should look. Which answer picture goes with the first bouquet of flowers?

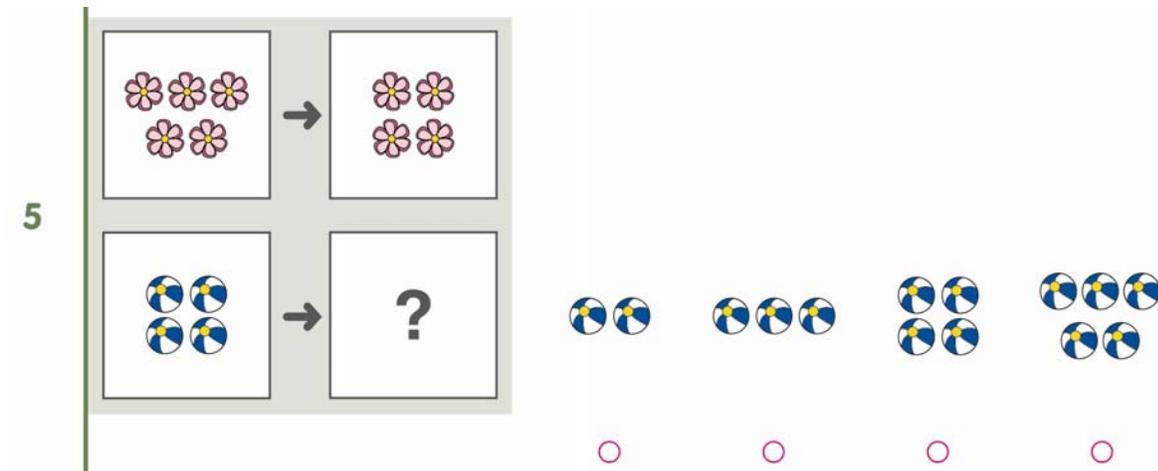
(Encourage responses. Then point to first answer picture as you **SAY**.)

First, one bouquet should change to two bouquets in the same way that one necklace changed to two necklaces. The six flowers in the first bouquet should be separated into one bouquet with two flowers and another bouquet with four flowers. Fill in the circle under the first answer picture to show that it is the correct answer.

(Check to make sure that all students have filled in the first circle.)

P5

Now look at the next practice question. Try to solve this practice question on your own.



Make sure students have enough time to solve the problem. Then **SAY**:)

Which answer picture goes in the box with the question mark?

(Encourage responses.)

How did you answer this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

There are five flowers in the first box and four flowers in the second box. What is the rule that changes five things into four things?

(Encourage responses.)

The rule is to take away, or subtract, one from the number of things in the first box.

(Point to the answer choices as you **SAY**:)

Which answer picture goes in the box with the question mark?

(Encourage responses.)

The answer should have three beach balls. Three beach balls are one less than four beach balls in the same way that four flowers are one less than five flowers. The second answer picture is the correct answer.

(Check to make sure that all students have filled in the second circle. If any students picked the fourth answer choice, **SAY**:)

Why is the last answer picture wrong? Five beach balls cannot be the answer because you must always go from left to right. Always apply the rule to the first box in the bottom row to decide what should go in the second box.

P6

Now look at the last practice question. Try to solve this practice question on your own.

(Make sure students have enough time to solve the problem. Then **SAY**!)

6

Which answer picture goes in the box with the question mark?

(Encourage responses.)

How did you answer this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

There are four stars in the first box and nine stars in the second box. What is the rule that changes four things into nine things?

(Encourage responses. If necessary, demonstrate by counting the stars.)

The rule is to add five to the number of things in the first box.

(Point to the answer choices as you **SAY**!)

Which answer picture goes in the box with the question mark?

(Encourage responses.)

The fourth answer picture with eight squares is correct. Five squares added to three squares makes eight squares just like five stars added to four stars makes nine stars.

(Check to make sure that students have filled in the fourth circle. If any students picked the second answer choice, **SAY**!)

*Be careful not to choose the second picture even though it looks similar to the correct answer.
You must count the boxes in each picture to make sure you get the right answer.*

Test 5: Number Puzzles, Level 8

Part 1: Overview of Number Puzzles

The Number Puzzles test requires the student to solve simple equations by choosing an answer that makes the numbers on both sides of the equal sign the same amount. To answer the questions in this test, the student must find the missing number that goes in the square with the question mark. This test measures understanding of mathematical identities.

In this test, students are asked to solve problems that look like this:

Which number should go in the square with the question mark?

$$\boxed{9} = \boxed{1} + \boxed{3} + \boxed{?}$$

4 5 6 7

When practicing the Number Puzzles questions, encourage students to use these strategies.

- Look at and calculate each side of the equal sign separately.
- Select the answer choice that makes the numbers on both sides of the equal sign total the same amount.

Students at this level tend to make the following common mistakes.

- Students might use the wrong arithmetic operation. For example, the student may add numbers instead of subtracting them.
- Students may choose an answer before finishing the calculation. For example, in the above sample question, the student might add the first two numbers, 1 plus 3, and then choose 4 as the answer.

Part 2: Number Puzzles Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in *blue italics*: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how to do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.

(Make sure each student has a practice booklet. Then **SAY**.)

*Open your practice booklet to page 3. You should be on the page with the **hammers** across the top.*

(Check that all students have the correct page.)

P1

Let's do the first practice question together.

(Hold up your copy of the practice booklet and point to the numbers and signs in the shaded rectangle as you **SAY**.)

1

4 = 2 + ?

2 3 4 6

Which number goes in the square with the question mark?

(Encourage responses.)

Both sides of the equal sign must total the same amount. The left side is 4. What number must we add to 2 so the right side of the equal sign also totals 4?

(Encourage responses.)

We must add 2 because 2 plus 2 is 4. Now the numbers on both sides of the equal sign total 4.

(Point to the answer choices as you **SAY**.)

Fill in the circle under the number 2 to show that it is the correct answer.

(Check to make sure that all students have filled in the first circle.)

P2

Let's do the next practice question.

2

5 = 1 + ?

3 4 5 6

Which number goes in the square with the question mark?

(Encourage responses.)

Both sides of the equal sign must total the same amount. The left side is 5. What number must we add to 1 to make the right side also total 5?

(Encourage responses.)

We must add 4 because 1 plus 4 is 5. Now the numbers on both sides of the equal sign total 5.

(Point to the answer choices as you **SAY**.)

Fill in the circle under the number 4 to show that it is the correct answer.

(Check to make sure that all students have filled in the second circle.)

P3

Let's do the next practice question.

3

3 = 7 - ?

4 5 6 10

Which number goes in the square with the question mark?

(Encourage responses.)

The left side of the equal sign is 3. What number must we subtract from 7 to make the right side of the question also total 3?

(Encourage responses.)

To find the missing number we must subtract 4. 7 minus 4 is 3. Now the numbers on both sides of the equal sign total 3.

(Point to the answer choices as you **SAY**.)

Fill in the circle under the number 4 to show that it is the correct answer.

(Check to make sure that all students have filled in the first circle.)

*Turn to the next page. You should be on the page with the **ducks** across the top.*

P4

Let's do the next practice question.

4	$\boxed{9} = \boxed{1} + \boxed{3} + \boxed{?}$			
	4	5	6	7
	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Which number goes in the square with the question mark?

(Encourage responses.)

The left side of the equal sign is 9. The right side of the equal sign shows 1 plus 3, which is 4. What number must we add to 4 to make the both sides of the equal sign total 9?

(Encourage responses.)

We must add 5 because 4 plus 5 is 9.

(Point to the answer choices as you **SAY**:)

Fill in the circle under the number 5 to show that it is the correct answer.

(Check to make sure that all students have filled in the second circle.)

P5

Look at the next practice question. Try to solve this practice question on your own.

5	$\boxed{8} = \boxed{6} + \boxed{5} - \boxed{?}$			
	11	4	3	2
	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

(Make sure students have enough time to solve the problem. Then **SAY**!)

Which number goes in the square with the question mark?

(Encourage responses.)

How did you answer this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

The left side of the equal sign is 8. The right side of the equal sign shows 6 plus 5, which is 11. What number must we subtract from 11 to make the both sides of the equal sign total 8?

(Encourage responses.)

We must subtract 3 because 11 minus 3 is 8.

(Point to the answer choices as you **SAY**!)

Fill in the circle under the number 3 to show that it is the correct answer.

(Check to make sure that all students have filled in the third circle.)

P6

Look at the next practice question. Try to solve this practice question on your own.

(Make sure students have enough time to solve the problem. Then **SAY**!)

6	$\boxed{2} = \boxed{11} - \boxed{7} - \boxed{?}$			
	9	6	4	2
	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Which number goes in the square with the question mark?

(Encourage responses.)

How did you answer this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

The left side of the equal sign is 2. The right side of the equal sign shows 11 minus 7, which is 4. What number must we subtract from 4 to make the right side of the equal sign 2 like the left side of the question?

(Encourage responses.)

We must subtract 2 because 4 minus 2 is 2.

(Point to the answer choices as you **SAY**.)

Fill in the circle under the number 2 to show that it is the correct answer.

(Check to make sure that all students have filled in the fourth circle.)

Test 6: Number Series, Level 8

Part 1: Overview of Number Series

The Number Series test requires the student to infer the next number in a series of beads on an abacus. It tests the ability to discover quantitative patterns and choose the simplest logical extension of the pattern from the choices available. These practice activities are designed to help students use and understand the logic and procedures necessary to solve the problems. Students who interpret the patterns of beads numerically rather than visually will find it easier to solve the more difficult questions.

In this test, students are asked to solve problems that look like this:

What is the pattern here?

Which one of these will come next?

When practicing the Number Series questions, encourage students to use these strategies.

- Think of (and say silently) a rule that describes the pattern in the strings of beads in the box. For example, each string has one more bead than the previous one.
- Apply the rule to determine which string of beads will come next in the series.
- Eliminate answer choices that do not fit the rule.

Students at this level tend to make the following common mistakes.

- Students may choose an answer that is the same as a string of beads in the initial series of beads rather than choosing the answer that continues the series.
- Students might choose an answer based on its appearance and not the actual count of beads. For example, the student may recognize the increasing pattern in the sample question above and select the fourth answer choice without counting the number of beads, simply because it has more beads than the strings in the box.
- Students may choose the answer choice that simply repeats the pattern. For example, in the sample question above, the student might choose the first answer choice.

Part 2: Number Series Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in *blue italics*: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how to do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.

(Make sure each student has a practice booklet. Then **SAY**.)

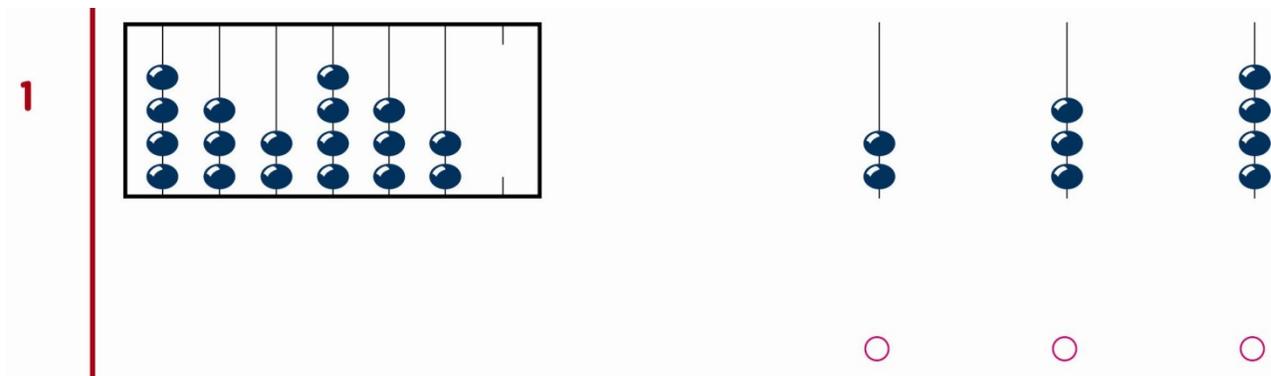
*Open your test booklet to 5. You should be on the page with the **rabbits** across the top.*

(Check that all students have the correct page.)

In these questions, you must look carefully at the beads in the box. Then choose the string of beads that comes next in the pattern.

P1

Let's do the first practice question together.



(Hold up your copy of the practice booklet and point to the beads in the box as you **SAY**.)

Look at the beads in the box. Let's count the beads on each string in the box.

(Count each string out loud.)

What pattern do they make?

(Encourage responses.)

There are 4 beads, then 3 beads, then 2 beads, then 4, 3, and 2 beads again. What is the pattern?

(Encourage responses.)

The pattern is 4, 3, 2 ... 4, 3, 2.

(Point to the empty space on the right side of the box as you **SAY**.)

How many beads should the next string have?

(Encourage responses.)

The next string should have 4 beads to repeat the pattern of 4, then 3, then 2 beads.

(Point to the answer choices as you **SAY**.)

Which answer picture shows this?

(Encourage responses.)

Fill in the circle under the string with 4 beads to show how the pattern continues.

(Check to make sure that all students have filled in the third circle.)

P2

Let's do the next practice question.

2

(Point to the beads in the box as you **SAY**.)

Let's count the beads on each string. What pattern do they make?

(Encourage responses. Explain that the horizontal line in the box and the answer choices separates the first five beads in a string from the remaining beads. It is there to help students count the longer strings of beads.)

There are 3 beads, then 4 beads, then 5 beads. How do the strings of beads change?

(Encourage responses.)

One bead is added to each new string.

(Point to the empty space on the right side of the box as you **SAY**.)

If one bead is added to each new string, then how many beads should come next?

(Encourage responses.)

Six beads follow 5 beads because 6 is 1 more than 5. Fill in the circle under the string with 6 beads to show that it is the correct answer.

(Check to make sure that all students have filled in the third circle. Then point to the first practice question as you **SAY**.)

Now look again at the first practice question. How does the pattern of beads in this second question differ from the pattern in the first question?

(Encourage responses. Then point to the beads in the first practice question.)

The pattern repeats 4, 3, 2 ... 4, 3, 2 in the first practice question.

(Point to the beads in the second practice question.)

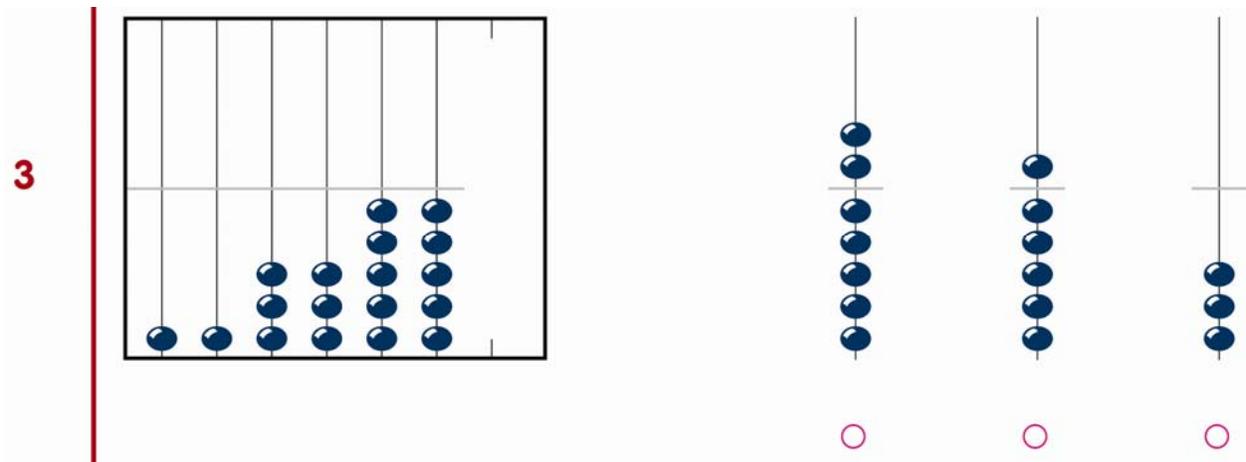
Do these beads also repeat a pattern?

(Encourage responses.)

No, the pattern does not repeat for this question. And so the correct answer is the string with 6 beads.

P3

Let's do the next practice question.



(Point to the beads in the box as you **SAY**.)

What is the pattern for this question?

(Encourage responses.)

The beads increase like they did in the last question, but in a different pattern. Let's count the beads on each string. There is 1 bead on the first string, 1 bead on the second string, 3 beads on the third string, 3 beads on the next string, then 5 beads, followed by another 5 beads. What is the pattern?

(Encourage responses.)

The pattern is 1, 1 ... 3, 3 ... 5, 5.

(Point to the empty space on the right side of the box as you **SAY**.)

How many beads should the next string have?

(Encourage responses.)

The number of beads is getting larger, so the next string should have more than 5 beads. Which strings in the answer choices have more than 5 beads?

(Encourage responses.)

The first answer choice has 7 beads and the second one has 6 beads. Which is the correct answer?

(Encourage responses.)

The string with 7 beads will follow the string with 5 beads because the pattern increases by 2. Fill in the circle under the string with 7 beads to show that it is the correct answer.

(Check to make sure that all students have filled in the first circle as you **SAY**.)

In this question, you must first see that the number of beads is increasing, but then you must also count **how many** new beads are added at each step. The second answer picture cannot be the correct answer because it is increasing by only one bead, not by two beads.

Turn to the next page. You should be on the page with the **t-shirts** across the top.

P4

Let's do the next practice question.

(Point to the beads in the box and the numbers below the box as you **SAY**!)

This question is just like the first three we solved except that now there are numbers below the strings of beads. Each number shows how many beads are on the string above it.

When you first look at the strings in the box, what does the pattern of beads seem to be?

(Encourage responses.)

The beads in this question look similar to the last question we just solved, but they have a different pattern. How is the pattern different?

(Encourage responses.)

The number of beads on each string gets smaller. Let's count the beads. There are 7 beads on the first string and 7 beads on the second string, 6 beads on the third string and 6 beads on the next string, then 5 beads, followed by another 5 beads. What is the pattern?

(Encourage responses.)

The pattern is 7, 7 ... 6, 6 ... 5, 5.

(Point to the empty space on the right side of the box as you **SAY**!)

How many beads should the next string have?

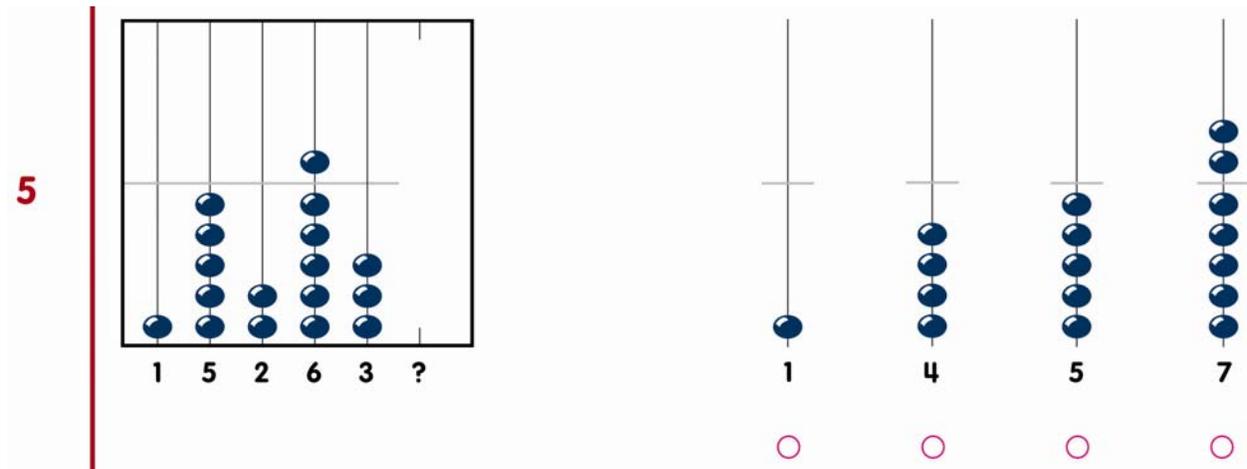
(Encourage responses.)

There should be 4 beads because the number gets smaller by 1. Fill in the circle under the string with 4 beads to show that it is the correct answer.

(Check to make sure that all students have filled in the third circle.)

P5

Let's do the next practice question.



(Point to the beads in the box as you **SAY**.)

Let's count the beads on each string in the box. There is 1 bead, then 5 beads, then 2 beads, then 6 beads, then 3 beads. What pattern do they make?

(Encourage responses.)

When you just look at the every other string, what is the pattern?

(Encourage responses. Then point to each string as you **SAY**.)

There is 1 bead on the first string, 2 beads on the third string, and 3 beads on the fifth string. So the number of beads is increasing 1, 2, 3.

(Point to the second and fourth strings as you **SAY**.)

What happens with the two longer strings?

(Encourage responses.)

There are 5 beads, then 6 beads. So the number of beads is increasing on those strings too.

(Point to the empty space on the right side of the box as you **SAY**.)

How many beads should the next string have?

(Encourage responses.)

The string with 7 beads is correct because the pattern increases by 1 on every other string. Fill in the circle under the string with 7 beads to show that it is the correct answer.

(Check to make sure that all students have filled in the fourth circle as you **SAY**.)

Let's look at this question again. Can you find any other way to solve it besides looking at every other string? There is more than one way to solve this question.

(Encourage responses.)

How can you make 1 bead into 5 beads?

(Encourage responses.)

Adding 4 will make 1 bead into 5 beads. Then how can you make 5 beads into 2 beads?

(Encourage responses.)

Subtracting 3 will make 5 beads into 2 beads. Then how can you make 2 beads into 6 beads?

(Encourage responses.)

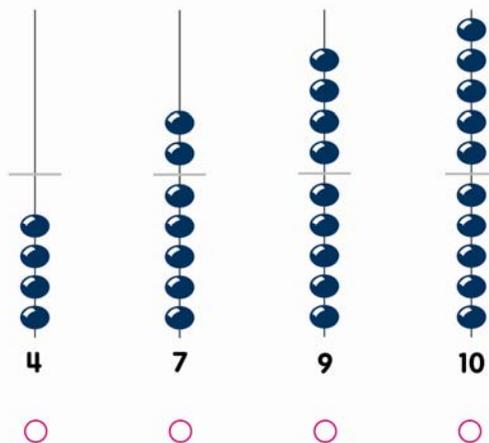
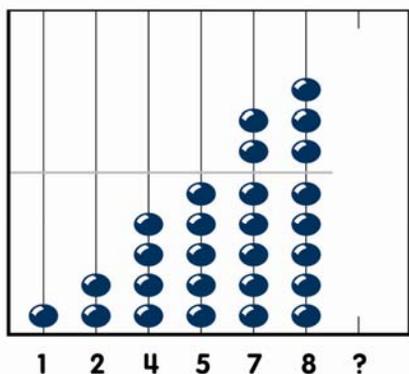
Adding 4 will make 2 beads into 6 beads. In the same way, subtracting 3 makes 6 beads into 3 beads. The new pattern goes like this: add 4, subtract 3, add 4, subtract 3, and so on. So the next step is to add 4 beads to the string with 3 beads. 4 plus 3 is 7. As you see, there can be different ways to find patterns and solve the problems.

(Check to make sure that students understand this reasoning.)

P6

Look at the last practice question. Try to solve this practice question on your own.

6



(Make sure students have enough time to solve the problem. Then **SAY**!)

How many beads should the next string have?

(Encourage responses.)

How did you answer this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

Let's count the beads on each string in the box. There is 1 bead, then 2 beads, then 4 beads, then 5 beads, then 7 beads, then 8 beads. What pattern do they make?

(Encourage responses.)

Look at the beads on each string. How does each string differ from the one before it?

(Encourage responses.)

The number of beads gets larger by 1, then 2, then 1, then 2, and then 1. So the pattern goes like this: add 1, add 2 ... add 1, add 2.

(Point to the empty space on the right side of the box as you **SAY**!)

How many beads should the next string have?

(Encourage responses.)

The next step is to add 2 beads to the string with 8 beads. 8 plus 2 is 10. So the string with 10 beads is the correct answer. Fill in the circle under the string with 10 beads to show how the pattern continues.

(Check to make sure that all students have filled in the fourth circle.)